1-6 Videos Guide

1-6a

- A brief review of inverse trigonometric functions

1-6b

- Calculus of inverse trigonometric functions
    - $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$ $\Longrightarrow$ $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$
    - $\frac{d}{dx}(\cos^{-1}x) = \frac{1}{-\sqrt{1-x^2}}$
    - $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{x^2+1}$ $\Longrightarrow$ $\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$
    - $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{x^2+1}$
    - $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$ $\Longrightarrow$ $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C$
    - $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$

Note: the formula for the derivative of the inverse secant assumes a restricted domain for the secant of $\{x|\ 0 < x < \pi/2 \text{ or } \pi < x < 3\pi/2\}$. If instead the restricted domain is $(0,\pi/2) \cup (\pi/2\,,\pi)$, then the differentiation rule becomes $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$.

1-6c

Exercises:

Find the derivative of the function. Simplify where possible.

- $g(x) = \arccos\sqrt{x}$
- $y = \cos^{-1}(\sin^{-1}t)$
- $R(t) = \arcsin(1/t)$

1-6d

- $y = \arctan\sqrt{\frac{1-x}{1+x}}$

1-6e

Evaluate the integral

- $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \frac{6}{\sqrt{1-p^2}}\,dp$
- $\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$